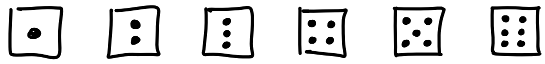
Basic Probability (Theoretical)

Theoretical Probability

Assuming all outcomes of an experiment are equally likely, the probability of an event E is:

$$P(E) = \frac{\boxed{}}{\boxed{}}$$

<u>Ex</u> Rolling a fair 6-sided die

Outcomes: 

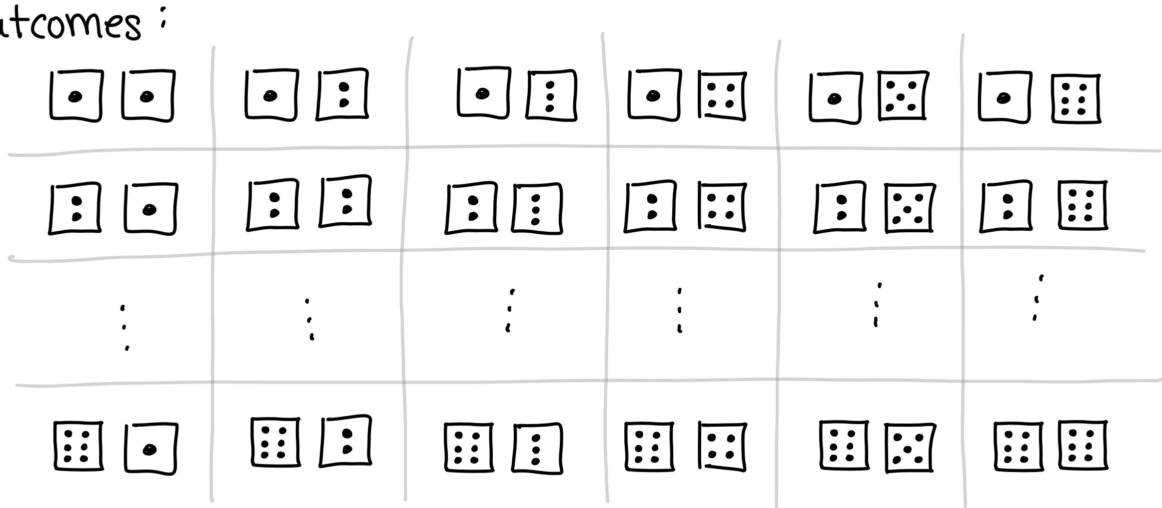
Probability of: rolling 2 is ____

not rolling 2 is ____

rolling 2 or 3 is ______

<u>Ex</u> Probability of rolling "doubles" when rolling two dice.

Outcomes:



P(doubles) = ________

<u>Ex</u> Flipping a fair coin twice

Outcomes: HH HT TH TT

Probability of getting heads twice: ____

Complementary events:

Event E, Complement E^c : NOT E

Ex E: Getting 2 heads in two coin flips

E^c: NOT getting 2 heads in two coin flips

$P(E) =$ ____________

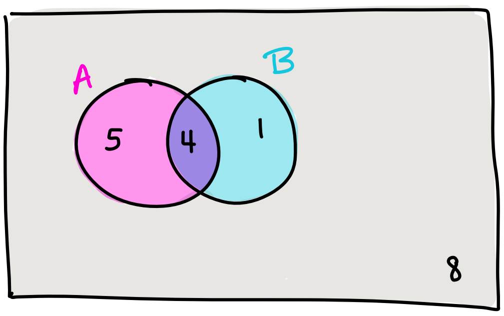
$P(E^c) =$ ____________

In general:

Connection b/w Complementary Events

$P(E^c) = \boxed{}$ (or $100\% - P(E)$)

"And", "Or"



$P(A) = \dfrac{\#A}{\text{Total}}$: 9 out of $\boxed{18}$

$P(B) = \dfrac{\#B}{\text{Total}}$: 5 out of $\boxed{18}$

$P(A \text{ and } B) = \dfrac{\#(A \text{ and } B)}{\text{Total}}$: 4 out of $\boxed{18}$

$P(A \text{ or } B) = \dfrac{\#(A \text{ or } B)}{\text{Total}}$: 10 out of $\boxed{18}$

Notice: 10 = 9 + 5 − 4

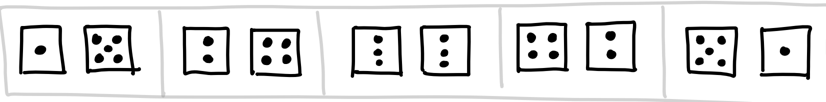
Connection b/w "And" and "Or"

$P(A \text{ or } B) = P(A) + P(B) - \boxed{}$

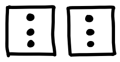
Ex Roll a pair of fair 6-sided dice

Probability of rolling a total of six or rolling doubles

A : Rolling doubles ; $P(A) = 6/36 = 1/6$

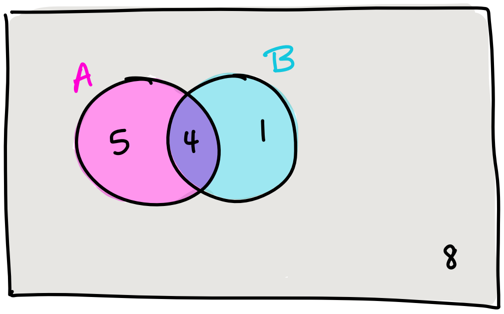
B : Rolling total of six : 

$P(B) = \boxed{}/36 = \boxed{}$

A and B :  $P(A \text{ and } B) =$ ____

$P(A \text{ or } B) =$ ____________ $=$ ________

Conditional Probabilities ("Given")



$$P(A \text{ given } B) = \frac{\#(A \text{ and } B)}{\# B}$$

4 out of 5

$$P(A \text{ given } B) \times P(B) = \frac{\#(A \text{ and } B)}{\# B} \times \frac{\# B}{\text{Total}} \qquad \left(\frac{4}{5} \times \frac{5}{18}\right)$$

$$= \frac{\#(A \text{ and } B)}{\text{Total}} \qquad \left(\frac{4}{18}\right)$$

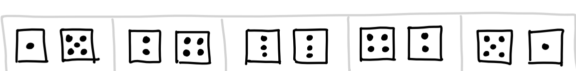
$$= P(A \text{ and } B)$$

Connection b/w "Given" and "And"

$$P(A \text{ given } B) \times P(B) = \boxed{}$$

Ex Rolling two fair six-sided dice

Prob. of rolling doubles (A), given rolled a total of six (B).



$P(A \text{ given } B) = \dfrac{\#(A \text{ and } B)}{\# B} = \boxed{}$

or $\dfrac{P(A \text{ and } B)}{P(B)} = \dfrac{\boxed{}}{\boxed{}} = \boxed{}$

Independent & Dependent Events

A : adult man in U.S. is taller than 6 feet
B : adult man in U.S. is in the NBA

Prob. of A vs. prob. of A given B
↳ not same : dependent events

A : flip heads on fair coin
B : today is payday

Prob. of A vs. prob. of A given B
↳ same : independent events

"And" Rule for Independent Events

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

when the likelihood of A occuring is the same, regardless of whether B occurs or not.

<u>Ex</u> Rolling two dice ; A : rolling doubles ; B: rolling total of six

$P(A \text{ and } B) = 1/36$ vs $P(A) \cdot P(B) =$ ____ $\cdot$ ____ $=$ ____

⇒ These two events are ____________.

From another perspective:

$P(A) =$ ____
vs
$P(A \text{ given } B) =$ ____

Likelihood of having rolled doubles ________ when we know that the total is six.

⇒ events are ____________.